A STEP FURTHER WITH THE PASCAL'S TRIANGLE

by Raymond O. Martin

The Pascal's triangle, as we know it, is a conspicuous triangle in the field of Mathematics. We use it everyday in our classes to find the coefficients of algebraic expressions in a simple and fast way. I will speak for myself in this, my education system taught me a mislead way to compute the values of the infinite triangle, and this planted questions in my mind. I am delighted that I am the one who has grown up to explain this misconception.

Take for instance the expression; $(a+b)^n$ *; to find the coefficients of this expression, draw the pascal's triangle, a depth of (n+1). Example;* $(a+b)^3$ *, the coefficients of this expression would be computed as;*

$$\begin{array}{ccccccc} & & & 1 & & & \\ & & 1 & & 1 & & \\ & 1 & & 2 & & 1 & \\ 1 & & 3 & & 3 & & 1 \end{array}$$

From the triangle, the final coefficients are given as;

$(a+b)^3 = 1a^3b^0 + a^2b^1 + a^1b^2 + a^0b^3$ *; simplified as;* $(a+b)^3 = a^3 + 3a^{2b} + 3ab^2 + b^3$

To obtain the coefficients is very simple. However, using the same approach to try and obtain the coefficients of another algebraic expression, specifically of the form; $(ax+b)^n$ *now things get out of hand, or so I thought at first. Studying the normal computation of the progressive values in the triangle, make some series of addition of previously input values, such that;*

$$\begin{array}{ccccccc} & & & 1 & & & \\ & & 1 & & 1 & & \\ & 1 & & (1+1) & & 1 & \\ 1 & & (1+2) & & (1+2) & & 1 \end{array}$$

This is how the system taught me to compute for the values of the pascal's triangle. However, I have pondered why such a popular and old formula can only find coefficients for 1-coefficient variables in expressions. I know that there is more to the triangle than I have been taught to use it, and so the search begins. Suppose I had an algebraic expression like; $(ax+b)^5$ *an example of an expression would be;* $(7x+4)^5$
Using the normal triangle, would immediately blow off. But, taking a different page on this computation can actually include this to be computable from the triangle. Suppose instead of always starting from an unquestionable point (1), I will now start from the values of a and b as;

$$a \quad b$$

I did not find perfect description of some properties in my calculations, so I will use rather what made sense to me, and of course some explanations along with it.
I will refer to the current value I will be computing as c.

7 4

Considering the fact that, from the original Pascal's triangle, the slopes were always 1, this means, *they had to be exponentiated, since that is what the triangle is meant for. I will do the same to my triangle, exponentiate each value on the edge. For me to have a complete and not an incomplete 2D pyramid at the top, I will populate the top of the triangle. Since the numbers are already on exponent 1, going further to the top will reduce the exponent factor, bringing it down to 0.*

1
7 4

The top value is a single number computed from 2 different co-relating numbers as; $7^0 = 1 \wedge 4^0 = 1$
With the same momentum pattern, the next series of edges after a and b on the left and the right are a^2 *and* b^2 *respectively. The number that comes beneath a and b is computed differently. Considering that a is smaller real number of the set {a, b}, c, is computed as the sum of the product of a and the sum of a and b and the product of the difference of a and b and the value that points to a or the computation can be calculated as the difference of the product of b and the sum of a and b and the product of the difference of a and b and the value that points to b. When the condition of a being less than b is reversed, then a is replaced with b in the computation formula and so is b with a. Note that according to this approach, the value of c and the reference pointing to the values of a and b are always changing. Moreover, the elements in the set used to add in the first part of the equation is relative to the depth into the triangle. In an equation, this is the approach;*

$$C_k = a \times \Sigma\{a, b\}_k + |(a - b)| \times a_k \Leftrightarrow a < b \wedge k > 0$$

So with this approach I will compute each and every value of the triangle defined by my starting values of a and b, in this case 4 and 7, till I get to my answer, in this case, the coordinates of the expanded expression. $n = 2$

$a = 4, b = 7, k = 1 \rightarrow a_k = 4, b_k = 7$

$C_1 = a \times \Sigma\{a_1, b_1\} + |(a + b)| \times a_1$

$C_1 = 4(7 + 4) + 3(4)$

$C_1 = 44 + 12$

$C_1 = 56$

When I fill the gap in the triangle;

1
7 4
49 56 16

The first values that I inserted into the triangle were the edges of the triangle, which are simply calculated as $a^{(k+1)}$ *and* $b^{(k+1)}$ *hence the values of 49 from* 7^2 *and 16 from* 4^2. *The value 56 was computed using the formula. In the same approach I will compute the next values for my coefficients.*

$$a = 4, b = 7 \rightarrow a_k = 56, b_k = 49$$

The coefficients changed and now we have $b_k < a_k$ *The value of a should not be confused with* a_k *the value of a is constant, it does not change, however,* a_k *changes according to k. I will now calculate the values of the rest of the Cs in this mini triangle.* $n = 3$

$C_2 = 4(49 + 56) + 3(56)$ $\quad$ $C_3 = 4(56 + 16) + 3(16)$

$C_2 = 420 + 168$ $\quad$ $C_3 = 288 + 48$

$C_2 = 588$ $\quad$ $C_3 = 336$

When I fill in the gaps;

1
7 4
49 56 16
343 588 336 64

The final answer from this calculation is;

$$(7x + 4)^3 = 343x^3 + 588x^2 + 336x + 64$$

This approach calculates the coefficients of an algebraic expression of the form $(ax + b)^n$. *The approach is efficient to see the changes on a small scale, however, on large scales, the* ***Binomial Expansion*** *method might be the go-to choice. This approach maintains consistency in using the Pascal's triangle when sorting for the coefficients of algebraic expression.*

Proof;

$(7x + 4)^3 = (7x + 4)^2(7x + 4)$

$(7x + 4)^3 = \big(7x(7x + 4) + 4(7x + 4)\big)(7x + 4)$

$(7x + 4)^3 = (49x^2 + 28x + 28x + 16)(7x + 4)$

$(7x + 4)^3 = 7x(49x^2 + 56x + 16) + 4(49x^2 + 56x + 16)$

$(7x + 4)^3 = 343x^3 + 392x^2 + 112x + 196x^2 + 224x + 64$

$(7x + 4)^3 = 343x^3 + (392x^2 + 196x^2) + (112x + 224x) + 64$

$(7x + 4)^3 = 343x^3 + 588x^2 + 336x + 64$